Published by Institute of Physics Publishing for SISSA

Received: *July 13, 2008*
Accepted: *September 6, 2008*
Published: *September 19, 2008*

Semi-classical strings in $AdS_4 \times CP^3$

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Abstract: In this paper, we study the semi-classical strings in $AdS_4 \times CP^3$ spacetime. We construct various kinds of string solutions, including the point-like, circular, folded and pulsating strings. For the circular and folded strings, we figure out their field theory dual operators. In particular, we discuss the anomalous dimensions of the corresponding operators from decoupled SU(2) $\times$ SU(2) spin chain. We find that in the large angular momentum limit, the field theory and string theory results are in very good agreement up to an interpolating function of coupling constant.

Keywords: AdS-CFT Correspondence, Long strings, Integrable Equations in Physics, Bethe Ansatz.

JHEP09(2008)096

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1. Introduction

The semi-classical strings has played an important role in studying AdS_5/SYM_4 correspondence. In a remarkable paper [1], it was found that the fluctuations around the point-like string captured the right string spectrum of BMN plane-wave background [2]. Moreover, the study of the multi-spin string solutions reveals that even when the configuration is far from BPS, the energy of the string solution with large angular momentum could be in perfect agreement with the ones calculated in dual gauge theory. The agreement beyond BPS limit relies on the fact that on string side the quantum corrections of strings was suppressed by the large quantum number, while on the field theory side the dual field operators to these string solutions are the composite operators, whose anomalous dimension matrix (ADM) could be related to the Hamiltonian of integrable spin chain. Moreover this agreement suggest that both sides of AdS_4/SYM_4 correspondence are integrable. There are lots of study on this topic. Please see [3] for a nice review and references.

Very recently, inspired by the study of Bagger-Lambert-Gustavson theory on N membranes [4 – 6], in [7] Aharony et.al. proposed a $\mathcal{N} = 6$ Chern-Simons theory coupled with bi-fundamental matter, describing N membrane on S^7/Z_k. In particular, it was pointed out that in the limit with $k \ll N \ll k^5$, this theory is dual to IIA string theory in $AdS_4 \times CP^3$. Many aspects of the BPS sector of this new AdS_4/CFT_3 correspondence have been studied in [7]. Quite recently, the near-BPS sector are also studied by using the Penrose limit of this IIA string theory background [8 – 10]. Other relevant work could be found in [12]–[25]. In particular, in [11, 9] it was pointed out there exist integrable spin-chain structure in the

field theory. It would be interesting to go beyond the near BPS sector and investigate the semiclassical configurations and their implications in AdS_4/CFT_3 correspondence.

In this paper, we will study the semi-classical string solutions in $AdS_4 \times CP^3$. We will mainly focus on the spinning solutions in CP^3. We will calculate their energy and angular momenta and compare with their field theory duals. This is possible due to the recent study of 2-loop integrable structure in the field theory [11, 9]. Our solutions include a point-like one which is BPS and corresponding to the ground states of the string theory in the IIA plan wave background obtained by taking Penrose limit. We also study the circular and folded string configurations. We find that some of our solutions are far from BPS, but still has a field theory dual. For the circular string and folded string, we discuss the energy of the corresponding operator from algebraic Bethe Ansatz equation(ABAE). The agreement between string theory and field theory results could be perfect if an interpolating function of coupling constant is introduced.

The paper will be organized as follows. In section 2, we will set up our system and have a general discussion of the semi-classical solutions. In section 3, we will present several kinds of spinning solutions. In section 4, we will discuss the field dual to some of these spinnning string solutions. We will end with conclusion and discussion in section 5.

2. Action and equations of motion

The three-dimensional $\mathcal{N} = 6$ superconformal theory proposed by ABJM in [7] is a Chern-Simons theory with gauge group $\mathrm{SU}(N) \times \mathrm{SU}(N)$ with bifundamental superfields A_1, A_2 and anti-bifundamental superfields B_1, B_2. The action has a pure Chern-Simons part:

$$S_{\mathrm{CS}} = \frac{k}{4\pi} \int \left(A_{(1)} \wedge dA_{(1)} + \frac{2}{3} A^3_{(1)} - A_{(2)} \wedge dA_{(2)} - \frac{2}{3} A^3_{(2)} \right), \tag{2.1}$$

and the superpotential

$$W = \frac{4\pi}{k} \mathrm{Tr}(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1). \tag{2.2}$$

Note that the sign before the Chern-Simons terms are opposite and the superpotential has actually an $\mathrm{SU}(2) \times \mathrm{SU}(2)$ global symmetry, which acting on the A's and the B's separately. The level of the Chern-Simons theory for the two components of the gauge group are k and $-k$, respectively. When k and N satisfy $k \ll N \ll k^5$, this field theory is dual to IIA superstring theory on $AdS_4 \times CP^3$ with constant dilaton, RR two-form and four-form fluxes. The constant dilaton reads:

$$e^{2\phi} = 2^{5/2} \pi N^{1/2} k^{-5/2}. \tag{2.3}$$

Since we are interested in semiclassical fundamental string solution in this background, the RR fluxes play no roles here.

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Let us start from the metric of $AdS_4 \times CP^3$,

$$\begin{aligned} ds^2 = & \frac{1}{4}R^2(-\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho(d\theta^2 + \sin^2\theta d\phi^2)) + R^2(d\xi^2 + \\ & \cos^2\xi\sin^2\xi\left(d\psi + \frac{1}{2}\cos\theta_1 d\varphi_1 - \frac{1}{2}\cos\theta_2 d\varphi_2\right)^2 + \frac{1}{4}\cos^2\xi(d\theta_1^2 + \sin^2\theta_1 d\varphi_1^2) + \\ & \frac{1}{4}\sin^2\xi(d\theta_2^2 + \sin^2\theta_2 d\varphi_2^2)). \end{aligned} \tag{2.4}$$

The radius R here is[1]

$$R = 2^{5/4}\pi^{1/2}\lambda^{1/4}, \tag{2.5}$$

where $\lambda \equiv N/k$ is the 't Hooft coupling constant of this three dimensional superconformal theory. In the above metric, the CP^3 part is the standard Fubini-Study metric which could be obtained in the following way. We first consider a unit S^7

$$\sum_{i=1}^{4}|Z_i|^2 = 1, \tag{2.6}$$

embedded in C^4. Then we parameterize the complex coordinates Z_i's as follows:

$$Z_1 = \cos\xi\cos\frac{\theta_1}{2}\exp\left[i\left(y + \frac{\psi + \varphi_1}{2}\right)\right], \tag{2.7}$$

$$Z_2 = \cos\xi\sin\frac{\theta_1}{2}\exp\left[i\left(y + \frac{\psi - \varphi_1}{2}\right)\right], \tag{2.8}$$

$$Z_3 = \sin\xi\cos\frac{\theta_2}{2}\exp\left[i\left(y + \frac{-\psi + \varphi_2}{2}\right)\right], \tag{2.9}$$

$$Z_4 = \sin\xi\sin\frac{\theta_2}{2}\exp\left[i\left(y + \frac{-\psi - \varphi_2}{2}\right)\right]. \tag{2.10}$$

Here $0 \leq \xi < \frac{\pi}{2}, 0 \leq y < 2\pi, -2\pi < \psi < 2\pi$ and (θ_i, φ_i) are coordinates of two S^2's. Now the induced metric on S^7 can be written as a U(1) fiber over CP^3:

$$ds^2_{S^7} = ds^2_{CP^3} + (dy + A)^2. \tag{2.11}$$

In this way, we get the metric on CP^3 as above (here A is a one-form).

For most of our solutions, we assume that $t, \phi, \psi, \varphi_1, \varphi_2$ are function of τ only and $\rho, \xi, \theta_1, \theta_2$ are only the periodic functions of σ. The bosonic part of the string Lagrangian is

$$L_B = \frac{1}{2}\sqrt{-g}g^{ab}G_{MN}\partial_a X^M \partial_b X^N \tag{2.12}$$

where G_{MN} is the background metric above and g_{ab} is the worldsheet metric. We can choose the conformal gauge, in which $g_{ab} = e^{\gamma}Diag(-1, 1)$. Then we have the action:

$$\begin{aligned} S = & \frac{\sqrt{\tilde{\lambda}}}{4\pi}\int d\sigma d\tau\left(\frac{1}{4}\cosh^2\rho\dot{t}^2 + \frac{1}{4}\rho'^2 + \frac{1}{4}\sinh^2\rho(\theta'^2 - \sin^2\theta\dot{\phi}^2) + \xi'^2\right. \\ & - \cos^2\xi\sin^2\xi\left(\dot{\psi} + \frac{1}{2}\cos\theta_1\dot{\varphi}_1 - \frac{1}{2}\cos\theta_2\dot{\varphi}_2\right)^2 + \frac{1}{4}\cos^2\xi(\theta_1'^2 - \sin^2\theta_1\dot{\varphi}_1^2) + \\ & \left.\frac{1}{4}\sin^2\xi(\theta_2'^2 - \sin^2\theta_2\dot{\varphi}_2^2)\right). \end{aligned} \tag{2.13}$$

[1]We take $\alpha' = 1$.

The relation between $\tilde{\lambda}$ and the 't Hooft coupling λ is:

$$\tilde{\lambda}^2 = 32\pi^2\lambda^2. \tag{2.14}$$

From the metric, we know that the background has at least five Killing vectors corresponding to the translations along $t, \phi, \psi, \varphi_1, \varphi_2$. The Killing vector along t gives the conserved energy and the other four Killing vectors give the conserved angular momenta. Let us make ansatz:

$$t = \kappa\tau, \quad \phi = v\tau, \quad \psi = \omega_1\tau, \quad \varphi_1 = \omega_2\tau, \quad \varphi_2 = \omega_3\tau \tag{2.15}$$

With this setup, we have the conserved quantities:

$$E = \frac{1}{4}\cosh^2\rho\sqrt{\tilde{\lambda}}\kappa \tag{2.16}$$

$$S = \sqrt{\tilde{\lambda}}\int\frac{d\sigma}{2\pi}v\sinh^2\rho\sin^2\theta \tag{2.17}$$

$$J_1 = \sqrt{\tilde{\lambda}}\int\frac{d\sigma}{2\pi}\cos^2\xi\sin^2\xi\left(\omega_1 + \frac{\cos\theta_1}{2}\omega_2 - \frac{\cos\theta_2}{2}\omega_3\right) \tag{2.18}$$

$$J_2 = \sqrt{\tilde{\lambda}}\int\frac{d\sigma}{2\pi}\left[\frac{1}{4}\cos^2\xi\sin^2\theta_1\omega_2 + \cos^2\xi\sin^2\xi\left(\frac{\cos^2\theta_1}{4}\omega_2 + \frac{\cos\theta_1}{2}\left(\omega_1 - \frac{\cos\theta_2}{2}\omega_3\right)\right)\right]$$

$$J_3 = \sqrt{\tilde{\lambda}}\int\frac{d\sigma}{2\pi}\left[\frac{1}{4}\sin^2\xi\sin^2\theta_2\omega_3 + \cos^2\xi\sin^2\xi\left(\frac{\cos^2\theta_2}{4}\omega_3 - \frac{\cos\theta_2}{2}\left(\omega_1 + \frac{\cos\theta_1}{2}\omega_2\right)\right)\right]$$

The equations of motion for $t, \phi, \psi, \varphi_1, \varphi_2$ are just the conservation of E, S, J_i's.

Now we turn to the equations of motion for $\rho, \theta, \xi, \theta_1, \theta_2$. The equations of motion are:

$$\rho'' = \sinh\rho\cosh\rho\dot{t}^2 - \sinh\rho\cosh\rho\sin^2\theta\dot{\phi}^2 + \sinh\rho\cosh\rho\theta'^2, \tag{2.19}$$

$$\frac{\partial}{\partial\sigma}\left(\frac{1}{2}\sinh^2\rho\theta'\right) = -\frac{1}{2}\dot{\phi}^2\sinh^2\rho\sin\theta\cos\theta, \tag{2.20}$$

$$\begin{aligned} 2\xi'' = &-\frac{1}{2}\sin 4\xi\left(\dot{\psi} + \frac{1}{2}\cos\theta_1\dot{\varphi}_1 - \frac{1}{2}\cos\theta_2\dot{\varphi}_2\right)^2 + \frac{1}{2}\cos\xi\sin\xi\sin^2\theta_1\dot{\varphi}_1^2 \\ &- \frac{1}{2}\sin\xi\cos\xi\sin^2\theta_2\dot{\varphi}_2^2 - \frac{1}{2}\sin\xi\cos\xi\theta_1'^2 + \frac{1}{2}\sin\xi\cos\xi\theta_2'^2, \end{aligned} \tag{2.21}$$

$$\begin{aligned} \frac{\partial}{\partial\sigma}\left(\frac{1}{2}\cos^2\xi\theta_1'\right) = &\cos^2\xi\sin^2\xi\sin\theta_1\left(\dot{\psi} + \frac{1}{2}\cos\theta_1\dot{\varphi}_1 - \frac{1}{2}\cos\theta_2\dot{\varphi}_2\right)\dot{\varphi}_1 \\ &- \frac{1}{2}\cos^2\xi\sin\theta_1\cos\theta_1\dot{\varphi}_1^2 \end{aligned} \tag{2.22}$$

$$\begin{aligned} \frac{\partial}{\partial\sigma}\left(\frac{1}{2}\cos^2\xi\theta_2'\right) = &-\cos^2\xi\sin^2\xi\sin\theta_2\left(\dot{\psi} + \frac{1}{2}\cos\theta_1\dot{\varphi}_1 - \frac{1}{2}\cos\theta_2\dot{\varphi}_2\right)\dot{\varphi}_2 \\ &- \frac{1}{2}\sin^2\xi\sin\theta_2\cos\theta_2\dot{\varphi}_2^2 \end{aligned} \tag{2.23}$$

The Virasoro constraint

$$\begin{aligned} G_{MN}(\partial_0X^M\partial_0X^N + \partial_1X^M\partial_1X^N) &= 0, \\ G_{MN}\partial_0X^M\partial_1X^N &= 0, \end{aligned} \tag{2.24}$$

gives

$$\begin{aligned}\cosh^2\rho\kappa^2/4 = &\frac{1}{4}(\rho'^2+\sinh^2\rho(\theta'^2+\sin^2\theta\dot{\phi}^2))\\ &+\xi'^2+\cos^2\xi\sin^2\xi\left(\omega_1+\frac{\cos\theta_1\omega_2-\cos\theta_2\omega_3}{2}\right)^2\\ &+\frac{1}{4}\cos^2\xi(\theta_1'^2+\sin^2\theta_1\omega_2^2)+\frac{1}{4}\sin^2\xi(\theta_2'^2+\sin^2\theta_2\omega_3^2)\end{aligned} \tag{2.25}$$

3. Various semi-classical string solution

In this section, we would like to discuss the semi-classical string solutions in $AdS_4\times CP^3$, with emphasis on the spinning string solutions in CP^3. For the string solutions in AdS_4, the discussion is very similar to the case in AdS_5. The only difference lies at the fact that there is only one spin quantum number in AdS_4 while there are two spins in AdS_5. The rotation string and pulsating string could be constructed easily.

For the semi-classical string in CP^3, the construction is different. In this case, ρ is constant and $t=\kappa\tau,\ ,\theta,\phi$ is constant, then the equation of motion for θ is satisfied and from the equation of motion for ρ, we get

$$\cosh\rho\sinh\rho\kappa=0, \tag{3.1}$$

which restricts $\rho=0$.

Compared to the similar construction of multi-spin solutions in $AdS_5\times S^5$ [26], the construction of spinning solutions in $AdS_4\times CP^3$ is more tricky and restrictive, whose dual field operators are also not transparent. We manage to find the dual operators for point-like string, a class of circular string and a class of folded string, by matching the global charges.

3.1 Point-like solution

Consider the point-like solution in which there is no dependence on σ.[2] Let $\theta_1=\theta_2=0$, then the equations of motion for θ_1 and θ_2 are satisfied. And from the equation of motion for ξ, we get

$$\sin 4\xi\left(\omega_1+\frac{1}{2}\omega_2-\frac{1}{2}\omega_3\right)^2=0 \tag{3.2}$$

So for generic ω_i's, ξ can only take $\pi/4$. The Virasoro constraints give

$$\kappa^2/4=\frac{1}{4}\left(\omega_1+\frac{\omega_2-\omega_3}{2}\right)^2 \tag{3.3}$$

Then we get

$$E=J_1=2J_2=-2J_3 \tag{3.4}$$

[2] After we finished our paper, a preprint [24] by Gromov and Vieria appears in arXiv. The discussions about point like strings there has some overlap with our discussions here.

Recall that J_i's is the quantum number corresponding to the Killing vector

$$J_1 = -i\frac{\partial}{\partial\psi} \tag{3.5}$$

$$J_2 = -i\frac{\partial}{\partial\varphi_1}, \quad J_3 = -i\frac{\partial}{\partial\varphi_2}. \tag{3.6}$$

The essential fact is that following [8], we can identify X_1, X_2, X_3, X_4 in (2.7)–(2.10) as the scalar fields $A_1, A_2, \bar{B}_1, \bar{B}_2$. Then we can write down the charges of the scalar fields as

$$J_1(A_1) = J_1(A_2) = J_1(B_1) = J_1(B_2) = 1/2 \tag{3.7}$$

$$J_2(A_1) = -J_2(A_2) = 1/2, \qquad J_2(B_1) = J_2(B_2) = 0 \tag{3.8}$$

$$J_3(A_1) = J_3(A_2) = 0, \qquad J_3(B_1) = -J_3(B_2) = -1/2 \tag{3.9}$$

Then the chiral operator $\mathrm{Tr}(A_1B_1)^J$ has $J_1 = J$, $J_2 = J/2$, $J_3 = -J/2$, $E = J$, in perfect match with the relation (3.4). In fact, as the case studied in [1], the point-like solution is dual to the chiral primary operators, which is BPS and can be identified as the ground state of the IIA string in the plane-wave background. The identification of $\mathrm{Tr}(A_1B_1)^J$ as the ground state has also been pointed out in [8]. Similar to the IIB string in $AdS_5 \times S^5$, it could be expected that the fluctuations around this point-like solution is actually the IIA string spectrum in the plane-wave background [1, 28, 8].

3.2 Folded string I

Let us try the following ansatz: $\theta_1 = \theta_2 = 0$, then we have

$$\xi'' = -\frac{1}{4}\sin 4\xi\tilde{\omega}^2 \tag{3.10}$$

where $\tilde{\omega} = \omega_1 + (\omega_2 - \omega_3)/2$. The Virasoro constraint is now

$$\frac{\kappa^2}{4} = \xi'^2 + \frac{\sin^2 2\xi}{4}\tilde{\omega}^2. \tag{3.11}$$

Since we consider the folded string here, ξ will take its maximal value at some ξ_0. When $\xi = \xi_0$, we have $\xi' = 0$ and so $\kappa^2 = \sin^2 2\xi_0\tilde{\omega}^2$. Then we get

$$\xi'^2 = \frac{\tilde{\omega}^2}{4}(\sin^2 2\xi_0 - \sin^2 2\xi), \tag{3.12}$$

which leads to

$$2\pi = 4\int_0^{\xi_0} \frac{2d\xi}{\tilde{\omega}(\sqrt{\sin^2 2\xi_0 - \sin^2 2\xi})}. \tag{3.13}$$

The angular momenta in this case is just

$$J_1 = \sqrt{\tilde{\lambda}}\int \frac{d\sigma}{2\pi}\cos^2\xi\sin^2\xi\tilde{\omega} \tag{3.14}$$

$$J_2 = \frac{J_1}{2} \tag{3.15}$$

$$J_3 = -J_2 \tag{3.16}$$

The eqs. (3.13) and (3.14) give us

$$\tilde{\omega} = \frac{2}{\pi}K(q), \tag{3.17}$$

$$J_1 = \frac{\sqrt{\tilde{\lambda}}}{2\pi}(K(q) - E(q)), \tag{3.18}$$

where $q = \sin^2 2\xi_0$ and $E(q), K(q)$ are the elliptic integrals of first kind and second kind, respectively. The energy E is

$$E = \frac{\sqrt{\tilde{\lambda}}}{4}\kappa = \frac{\sqrt{\tilde{\lambda}}}{4}\tilde{\omega}\sin 2\xi_0 = \frac{\sqrt{\tilde{\lambda}}}{2\pi}\sin 2\xi_0 K(q). \tag{3.19}$$

This string configuration is a folded string. The relation between different angular momenta suggest that the solution has only one angular momentum. It is reminiscent of the folded string solution spinning in S^5 discussed in [1]. It is interesting to consider the large J limit, which corresponds to $\xi_0 \to \pi/4$. In this limit, one has $E \to \infty$, $J_1 \to \infty$ and $E - J_1 \sim \frac{\sqrt{\tilde{\lambda}}}{2\pi}$. This looks similar to the relation in the giant magnon case with $p = 1$ in [9], but actually folded string solution is very different from the semiclassical string of giant magnon explicitly constructed in [10, 22].

3.3 Circular strings

In this subsection, we will fix $\xi = \xi_0$ being a constant. First let us further assume that $\omega_3 = 0$, $\theta_1 = 0$, the equation of motion for θ_1 is satisfied. From the equation of motion for θ_2 we have

$$\theta_2'' = 0, \tag{3.20}$$

so $\theta_2 = n\sigma$, (we choose $n \neq 0$). The equation of motion for ξ gives:

$$\cos 2\xi_0 = -\frac{n^2}{(2\omega_1 + \omega_2)^2}. \tag{3.21}$$

For this solution, we have:

$$J_1 = \sqrt{\tilde{\lambda}}\cos^2\xi_0 \sin^2\xi_0 \left(\omega_1 + \frac{1}{2}\omega_2\right) = 2J_2, J_3 = 0, \tag{3.22}$$

and

$$E^2 = \frac{J_1^2}{4\sin^2\xi_0\cos^2\xi_0} + \frac{\tilde{\lambda}\sin^2\xi_0 n^2}{16}. \tag{3.23}$$

This is a string solution with one independent angular momentum. For $n = 0$, we just come back to the point-like solution discussed before.

We can also make ansatz that $\omega_2 = \omega_3 = 0$, then the equations of motion for θ_1, θ_2 give

$$\theta_1'' = \theta_2'' = 0, \tag{3.24}$$

which gives $\theta_i = n_i\sigma$. If we choose n_1 and n_2 to be nonzero, then $J_2 = J_3 = 0$, The equation of motion for ξ gives:

$$\cos 2\xi_0 = \frac{n_1^2 - n_2^2}{4\omega_1^2}. \tag{3.25}$$

The relation between E and J_1 is

$$E^2 = \frac{J_1^2}{4\sin^2\xi_0\cos^2\xi_0} + \frac{\tilde{\lambda}}{16}(\cos^2\xi_0 n_1^2 + \sin^2\xi_0 n_2^2). \tag{3.26}$$

This solution is a circular string with one angular momentum. Especially if we choose $n_1 = \pm n_2$, we have $\xi_0 = \pi/4$ and

$$E = \sqrt{J_1^2 + \frac{\tilde{\lambda}n_1^2}{16}} = J_1\left(1 + \frac{\tilde{\lambda}n_1^2}{32J_1^2} + \cdots\right). \tag{3.27}$$

In this case, the circular string has a field theory dual. Let us consider the composite operator in field theory $\mathrm{Tr}((A_1B_1)^J(A_2B_2)^J)$. It has $J_1 = 2J$, $J_2 = J_3 = 0$ and at the classical level, $E = 2J$. This is in consistent with the relations that the circular string respect to. We will study this operator in the next section.

3.4 Folded string II

The study of the circular string solutions suggest that one may have to fix $\xi = \pi/4$ in order to find their field theory dual operator chains. In this case, from the equation of motion for ξ, it is quite natural to require $\theta_1 = \pm\theta_2$. Actually, this is the only possible way to have nontrivial solution. Then from the equations for θ_1 and θ_2, we find that this is only possible for $\omega_2 = -\omega_3$ and

$$\theta_1'' = \sin\theta_1\omega_1\omega_2. \tag{3.28}$$

This equation is in consistency with the Virasoro constraint, which has

$$\kappa^2 = \theta_1'^2 + \omega_1^2 + \omega_2^2 + 2\omega_1\omega_2\cos\theta_1. \tag{3.29}$$

Let us first consider two special case. When we take $\omega_2 = 0$, this reduce to the circular string we studied before.

However, the case with $\omega_1 = 0$ is also interesting. In this case, we have

$$J_1 = 0, J_2 = -J_3 = \frac{\sqrt{\tilde{\lambda}}}{8}\omega_2, \kappa^2 = \omega_2^2 + n^2 \tag{3.30}$$

This is another circular string solution, quite similar to the first one, but the field theory dual is very different. To respect the relation between quantum numbers, we are led to considering the following operators: $\mathrm{Tr}(A_1B_1)^{J_2/2}(B_2^\dagger A_2^\dagger)^{J_2/2}$. The energy of string in the large J_2 limit is

$$E = 2J_2 + \frac{n^2\tilde{\lambda}}{64J_2} + \cdots. \tag{3.31}$$

Classically the dual operator has dimension $\Delta = 2J_2$, in consistent with the zero order string energy. It should be keep in mind that the first order correction in the string energy is of order $\tilde{\lambda}/J_2^2$, similar to the circular string we discussed before.

In general, the solution is a folded string configuration. Since θ_1 is periodic of σ, so we have

$$-\theta_1(0) \le \theta_1(\sigma) \le \theta_1(0), \kappa^2 = \omega_1^2 + \omega_2^2 + 2\omega_1\omega_2\cos\theta_1(0). \tag{3.32}$$

If $\omega_1\omega_2 > 0$, θ is varying around π. On the contrary, if $\omega_1\omega_2 < 0$, the folded string is centered at $\theta_1 = 0$. Without losing generality, we will assume $\omega_1 > 0, \omega_2 < 0$ such that $\omega_1\omega_2 < 0$. Then, we have

$$2\pi = \int_0^{2\pi} d\sigma = \frac{2}{\sqrt{-\omega_1\omega_2}} \int_0^{\theta_1(0)} \frac{d\theta_1}{\sqrt{\sin^2\frac{\theta_1(0)}{2} - \sin^2\frac{\theta_1}{2}}}. \tag{3.33}$$

This gives us

$$\sqrt{-\omega_1\omega_2} = \frac{2}{\pi}K(x), \tag{3.34}$$

where $x = \sin^2\frac{\theta_1(0)}{2}$.

The energy of the folded string solution is just $E = \frac{1}{4}\sqrt{\tilde{\lambda}}\kappa$. The angular momenta are

$$\begin{aligned} J_1 &= \sqrt{\tilde{\lambda}} \int \frac{d\sigma}{2\pi}\frac{1}{4}(\omega_1 + \cos\theta_1\omega_2) \\ &= \frac{\sqrt{\tilde{\lambda}}}{2\pi\sqrt{-\omega_1\omega_2}} \left((\omega_1 - \omega_2)K(x) + 2\omega_2 E(x)\right), \end{aligned} \tag{3.35}$$

$$J_2 = \frac{\sqrt{\tilde{\lambda}}}{8\pi\sqrt{-\omega_1\omega_2}} \int_0^{\theta_1(0)} \frac{(\omega_2 + \omega_1\cos\theta_1)d\theta_1}{\sqrt{\sin^2\frac{\theta_1(0)}{2} - \sin^2\frac{\theta_1}{2}}} \tag{3.36}$$

$$= \frac{\sqrt{\tilde{\lambda}}}{4\sqrt{-\omega_1\omega_2}} \left((\omega_2 - \omega_1)K(x) + 2\omega_1 E(x)\right), \tag{3.37}$$

$$J_3 = -J_2. \tag{3.38}$$

We have the following relation

$$\frac{\omega_1}{2}J_1 - \omega_2 J_2 = \frac{\sqrt{\tilde{\lambda}}}{8}(\omega_1^2 - \omega_2^2) \tag{3.39}$$

In the case that $\omega_1 = -\omega_2$, we have $J_1 = -2J_2$.

It is convenient to introduce the following quantities:

$$\mathcal{E} = \frac{E}{\sqrt{\tilde{\lambda}}}, \quad \mathcal{J}_i = \frac{E}{\sqrt{\tilde{\lambda}}}, \quad i = 1, 2, 3. \tag{3.40}$$

In terms of these quantities, we have

$$\omega_1 = K(x)\frac{K(x)\mathcal{J}_1 - 2\mathcal{J}_2(2E(x) - K(x))}{E(x)(K(x) - E(x))} \tag{3.41}$$

$$\omega_2 = K(x)\frac{2K(x)\mathcal{J}_2 - \mathcal{J}_1(2E(x) - K(x))}{E(x)(K(x) - E(x))}, \tag{3.42}$$

and the following key relations

$$\left(\frac{\mathcal{E}}{K(x)}\right)^2 - \left(\frac{\mathcal{J}_1 + 2\mathcal{J}_2}{E(x)}\right)^2 = \frac{4}{\pi^2}x \tag{3.43}$$

$$\left(\frac{\mathcal{J}_1 - 2\mathcal{J}_2}{E(x) - K(x)}\right)^2 - \left(\frac{\mathcal{J}_1 + 2\mathcal{J}_2}{E(x)}\right)^2 = \frac{4}{\pi^2}. \tag{3.44}$$

We will show that in the next section, due to the above relation the folded string could be in perfect match with the dual field theory operators up to an interpolating function.

The dual operators in the field theory is somehow subtle. To match the above angular momenta, we propose the following identification:

$$\begin{cases} \mathrm{Tr}((A_1B_1)^{\frac{J_1}{2}+J_2}(A_2B_2)^{\frac{J_1}{2}-J_2}), & \text{for } \omega_1+\omega_2>0 \\ \mathrm{Tr}((B_1^\dagger A_1^\dagger)^{-(\frac{J_1}{2}+J_2)}(A_2B_2)^{\frac{J_1}{2}-J_2}), & \text{for } \omega_1+\omega_2<0 \end{cases} \tag{3.45}$$

However, if we take $\omega_1=-\omega_2$, then the above operators reduce to $\mathrm{Tr}(A_2B_2)^{J_1}$, which is a BPS primary and has $\Delta=J_1$ without quantum correction. But from the string calculation we know that there do exist higher order corrections.

3.5 Pulsating string

Before ending this section, let us discuss pulsating string, another kind of semi-classical string solution. The pulsating string purely in AdS_4 is quite similar to the one in AdS_5 [27]. So we focus on the circular pulsating string expanding and contracting on CP^3. To simplify the discussion, we let $\theta,\phi,\theta_1,\theta_2,\varphi_1,\varphi_2$ be fixed to zero, $t=\kappa\tau,\psi=n\sigma$ and ρ,ξ be the function of τ. Then the Green-Schwarz action is

$$S=\frac{\sqrt{\tilde{\lambda}}}{4\pi}\int d\sigma d\tau\left(\frac{\kappa^2}{4}\cosh^2\rho-\frac{1}{4}\dot{\rho}^2-\dot{\xi}^2+\cos^2\xi\sin^2\xi n^2\right) \tag{3.46}$$

which leads to the equations of motion

$$\ddot{\rho}=-\kappa^2\sinh\rho\cosh\rho \tag{3.47}$$

$$\ddot{\xi}=-\frac{1}{4}n^2\sin 4\xi \tag{3.48}$$

On the other hand, the Virasoro constraint is now

$$\frac{1}{4}(-\kappa^2\cosh^2\rho+\dot{\rho}^2)+\dot{\xi}^2+\cos^2\xi\sin^2\xi n^2=0 \tag{3.49}$$

To be consistent with the equations of motion, we notice that $\dot{\rho}$ has to be vanishing. So we fix $\rho=0$. Then we have only one equation to solve. Let $\eta=2\xi$, we have

$$\dot{\eta}^2+\sin^2\eta n^2-\kappa^2=0 \tag{3.50}$$

or

$$\ddot{\eta}=-\frac{n^2}{2}\sin 2\eta. \tag{3.51}$$

This looks nice a one-dimensional pendulum. From eq. (3.50), we know that when η take the maximal value, η_0, $\dot{\eta}=0$. Then $\kappa^2=n^2\sin^2\eta_0$. So

$$\dot{\eta}^2=n^2(\sin^2\eta_0-\sin^2\eta). \tag{3.52}$$

If the period of this pendulum is T (this is measure by the worldsheet time τ), we have,

$$T=4\int_0^{\eta_0}\frac{d\eta}{n\sqrt{\sin^2\eta_0-\sin^2\eta}}. \tag{3.53}$$

When $\tau\in[0,\frac{\pi}{4}]$, we have

$$\tau=\int_0^{\eta}\frac{d\tilde{\eta}}{n\sqrt{\sin^2\eta_0-\sin^2\tilde{\eta}}}. \tag{3.54}$$

4. Field theory dual operators

The integrable spin chain in ABJM theory has been discussed in [11, 9]. It was pointed out in [11], at two-loop order, the anomalous dimension matrix (ADM) of the composite operators constructed from the scalar fields could be identified with an integrable Hamiltonian of an SU(4) spin chain. Including the fermions in the operators, the spin chain is extended to SU(2|2) in [9]. More precisely, following the notation in [11] let us consider the following gauge invariant operators of the form

$$\mathrm{Tr}(Y^{A_1}Y^\dagger_{B_1}Y^{A_2}Y^\dagger_{B_2}\cdots Y^{A_L}Y^\dagger_{B_L}), \tag{4.1}$$

where

$$Y^A=(A_1,A_2,B_1^\dagger,B_2^\dagger),\qquad Y^\dagger_A=(A_1^\dagger,A_2^\dagger,B_1,B_2). \tag{4.2}$$

In general, the leading order ADM of this class of composite operators can be identified with the Hamiltonian of an SU(4) spin chain with sites alternating between the fundamental and anti-fundamental representations. We will not review the relevant discussions here. There are three sets of Bethe roots, satisfying the coupled Bethe equations.

Let us consider SU(2) × SU(2) subsector of the SU(4) spin chain, in which Y^A take only A_1, A_2 and $Y^\dagger_A$ take only B_1, B_2. Obviously the operator dual to the circular string and the folded string ($\omega_1+\omega_2>0$ case) belong to this subsector. In this case, we have two decoupled SU(2) chains. The middle Bethe roots r_j in [11] will not appear in this case. And the Bethe equations are:

$$\left(\frac{u_j+i/2}{u_j-i/2}\right)^L=\prod_{k=1,k\neq j}^{M_u}\frac{u_j-u_k+i}{u_j-u_k-i}, \tag{4.3}$$

$$\left(\frac{v_j+i/2}{v_j-i/2}\right)^L=\prod_{k=1,k\neq j}^{M_v}\frac{v_j-v_k+i}{v_j-v_k-i}. \tag{4.4}$$

The trace condition is

$$1=\prod_{j=1}^{M_u}\frac{u_j+1/2}{u_j-1/2}\prod_{j=1}^{M_v}\frac{v_j+1/2}{v_j-1/2}. \tag{4.5}$$

The energy is

$$E=\lambda^2\left(\sum_{j=1}^{M_u}\frac{1}{u_j^2+\frac{1}{4}}+\sum_{j=1}^{M_v}\frac{1}{v_j^2+\frac{1}{4}}\right). \tag{4.6}$$

Here we have $L=2J$ and $M_u=M_v=L/2$.

One choice to satisfy the trace condition is to satisfy

$$1=\prod_{j=1}^{M_u}\frac{u_j+1/2}{u_j-1/2}, \tag{4.7}$$

and

$$1=\prod_{j=1}^{M_v}\frac{v_j+1/2}{v_j-1/2}, \tag{4.8}$$

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separately. Then these two chains are totally unrelated.

For one chain in the large J limit, the Bethe roots may distribute in two different ways, as discussed in [29]. For the circular string, the Bethe roots distribute along the imaginary axis. It is natural to expect that for the circular string in this paper, the Bethe roots distribution follow the same way. Then use the results in [29], we get that in the large N and large J limit, the anomalous dimension of this operator:

$$\gamma = \frac{4\pi^2\lambda^2}{J}. \tag{4.9}$$

So the dimension of this operator obtained from the field theory side at the weak coupling is:

$$\begin{aligned}\Delta &= 2J + \frac{4\pi^2\lambda^2}{J} + \cdots \\ &= J_1\left(1 + \frac{\tilde{\lambda}^2}{4J_1^2} + \cdots\right).\end{aligned} \tag{4.10}$$

Comparing this result with the string energy (3.27), we find that at large J limit, the energy is proportional to J_1 at the leading order and the first order correction is always $1/J_1^2$. However, on string side the first order contribution is linear in $\tilde{\lambda}$, while on the field theory side, it is quadratic in $\tilde{\lambda}$. This is very similar to what happens in the point-like string case, where the string and field theory result on fluctuation spectrum has a mismatch of factor λ.

The folded string case is more impressive. Let us just consider the dual operators $\mathrm{Tr}((A_1B_1)^{\frac{J_1}{2}+J_2}(A_2B_2)^{\frac{J_1}{2}-J_2})$, which belong to the decoupled subsector. For a single subsector, the operator looks like the same one corresponding to the IIB folded string in $AdS_5 \times S^5$ with two angular momenta, namely the operators of the form $\mathrm{Tr}Z^{J_1}\Phi^{J_2} + \cdots$. In IIB case, the match between the string and the field theory result is in a highly non-trivial way [30]. Without getting into details, we can show that our folded string is also in good match with the field theory result, up to an interpolating functin of the 't Hooft coupling constant. Notice that the relation (3.43) is the same as the relation (2.2) in [30], after identifying the angular momenta properly. And since we have the similar integrable structure, the discussion in [30] could be applied to our case straightforwardly.[3] In this way, we show that for the folded string, we have very good first leading order match in AdS_4/CFT_3 correspondence.

For other circular string and folded string, the study of the dual operators is quite similar. the dual operators should belong to the decoupled $\mathrm{SU}(2) \times \mathrm{SU}(2)$ subsector. And the relation between energy and the angular momenta in these cases is consisent with this statement.

5. Conclusion

In this paper, we studied the semi-classical string configurations in $AdS_4 \times CP^3$ and their possible field theory dual. We constructed point-like, circular, folded and pulsating strings

[3]The only difference is that now the first non-trivial correction in the field theory appears at the order of λ^2, so an interpolating fuction of λ is needed here. See also our discussions in the next section.

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in CP^3 and calculate their energy and angular momenta. For the circular strings and one class of folded string configurations, we figured out their field theory dual operators. For one class of circular string, the dual operators fall into the SU(2) $\times$ SU(2) subsector of an integrable SU(4) spin chain. In this subsector, the fact that the two spin chains decouple allow us to calculate the eigenvalues of ADM from Bethe equations. On the field side, the ADM get correction only at two-loop order, which is proportional to λ^2. This suggest that in the large angular momentum limit, an effective expansion parameter could be λ^2/J^2. On the other hand, from the string calculation, we learn that the first order correction is linear in the expansion parameter λ/J^2. As suggested in [8 – 10], there should exist a interpolating function $f(\lambda)$ which approaches λ at weak coupling and $\sqrt{\lambda}$ at strong coupling. Combining the result we found in this paper, we suggest that in the study of the spinning strings, the effective expansion parameter is $f^2(\lambda)/J^2$. The nontrivial functional match in the first leading order in the folded string case gives very strong support to this suggestion.

Acknowledgments

The work was partially supported by NSFC Grant No.10535060,10775002 and NKBRPC (No. 2006CB805905). JB would like to thank Matteo Bertolini, Bo Feng, Edi Gava, Chethan Gowdigere, Mahdi Torabian and Ho-Ung Yee for helpful discussions.

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